

MAT 534: HOMEWORK 5

DUE W Sep 30

Problems marked by asterisk (*) are optional.

1. Let G be abelian group, which is generated by x, y, z with relations $x^3 = xy^2z^3 = 1$. Represent G as a product of cyclic groups.
Hint: You can write G additively and work out Smith normal form.
2. Same question for the group $\mathbb{Z}^3/L$, where L is generated by $u_1 = (3, 2, 5)$, $u_2 = (0, 1, 3)$, $u_3 = (0, 1, 5)$.
3. Let A be $n \times n$ matrix with integer coefficients, and let $L \leq \mathbb{Z}^n$ be the subgroup generated by the rows of A . Prove that $\mathbb{Z}^n/L$ is infinite if $\det A = 0$ and if $\det A \neq 0$, $|\mathbb{Z}^n/L| = |\det A|$. (You can use existence of Smith normal form).
- *4. From Dummit and Foote, problems 2 and 3 on p. 165.
5. From Dummit and Foote, problem 8 on p. 166.
- *6. From Dummit and Foote, problem 14 on p. 167.